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# Delivering Endogenous Inertia in Prices and Output

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## **Abstract**

This paper presents a DGE model in which aggregate price level inertia is generated endogenously by the optimizing behaviour of price setting firms. All the usual sources of inertia are absent here ie., all firms are simultaneously free to change their price once every period and face no adjustment costs in doing so. Despite this, the model generates persistent movements in aggregate output and inflation in response to a nominal shock. Two modifications of a standard one-quarter pre-set price model deliver these results: learning-by-doing and habit formation in leisure.

Key words: Endogenous price stickiness, Business Cycles, Inflation, Nominal rigidities, Learning-by-doing, Habit formation, Propagation mechanisms, Persistence.

# 1 Introduction

There has been a recent surge in interest in dynamic general equilibrium models in which firms adjust their prices infrequently. Many of these models employ one of two classes of time-dependent pricing rules associated with Taylor (1999) and Calvo (1983).<sup>1</sup> In the former, prices are set for a given number of periods and the opportunity to adjust prices is staggered so that not all firms can adjust prices in any given period. In the latter, firms face a fixed probability of being able to adjust prices in each period. While the duration for which prices are fixed is uncertain for an individual firm, the average duration is known and in the aggregate a constant fraction of all firms will adjust prices every period as in the Taylor model.<sup>2</sup>

While these models have had some success in generating empirically plausible business cycles in response to monetary shocks the pricing arrangements embedded in the models are theoretically unappealing.<sup>3</sup> This theoretical weakness arises from the exogenous nature of the pricing arrangements imposed upon firms which determine both the length of time for which prices cannot be re-optimized as well as the degree of synchronization among firms. This can have important consequences for the ability of these models to predict the response of the economy to changes in the economic environment, especially to changes in monetary policy. While one might expect that the optimal pricing arrangements of firms may respond to policy, they cannot in the model. Since the duration of price stickiness and the degree of staggering of pricing decisions influence the response of aggregate variables in the model one may not end up with sensible predictions regarding how the economy will respond to these changes.

Staggered price setting models were popular despite this well understood weakness because staggering was viewed as a critical element, along with long periods of price stickiness, in generating an inertial response of the price level and aggregate output to monetary shocks. However recent work has questioned the centrality of these two phenomena in propagating nominal shocks. Chari, Kehoe and McGrattan (2000) forcefully argue that staggering

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<sup>1</sup>There are too many sticky price dynamic general equilibrium models to list here. Some examples are King and Watson (1996), Yun (1996), Cho et al (1997), Chari et al. (2000), Bergin and Feenstra (2000), Erceg et al. (2000) and Huang and Liu (2001).

<sup>2</sup>See Ascari (2004) for a reason to prefer Taylor style models in the presence of trend inflation.

<sup>3</sup>See Christiano, Eichenbaum and Evans (2005) for example.

of pricing decisions is ineffective in propagating output beyond the assumed duration for which prices are fixed. In addition, Christiano, Eichenbaum and Evans (2005) show that price staggering is not crucial to generating realistic impulse responses. Finally Bils and Klenow (2004) show that on average prices change much more quickly in US data than has been assumed in the sticky price literature.<sup>4</sup>

The goal of this paper is to show that it is not necessary to retain either unappealing element in order to generate inertia not only in output but also in the aggregate price level. To make this point forcefully, the paper restricts the amount of exogenous price rigidity to one period, i.e., all firms set prices simultaneously at the beginning of each period. None the less, the aggregate price level will adjust slowly to a money growth shock because all firms optimally choose to adjust their prices slowly<sup>5</sup>. In the standard one-period sticky price model, firms wish to adjust prices in proportion with the expected change in marginal costs next period. One way to slow down the adjustment of prices is to introduce mechanisms that prevent marginal costs from rising too fast. The other way is for firms to actually choose to adjust prices *less* than proportionally to expected marginal cost. In other words, firms must change their markup. This paper incorporates both of the above features.<sup>6</sup> As a result the model can generate prices that adjust very slowly. Since the firm could in principle adjust fully to expected future inflation after the first period, any subsequent sluggishness seen in the impulse response of prices is endogenous. Indeed, in the standard one period sticky price model, almost all of the adjustment in a firm's price occurs in the first period after the shock.

The paper uses two mechanisms that build upon each other to quantitatively generate realistic persistence in inflation and output. The first mechanism modifies the technological environment in which firms operate by introducing learning-by-doing.<sup>7</sup> The second mechanism modifies consumers

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<sup>4</sup>Bils and Klenow suggest that half of the prices studied lasted no more than 4.3 months which is much shorter than the assumed duration of price rigidity (typically about 12 months) in the literature.

<sup>5</sup>This is not induced by imposing menu costs on the firms.

<sup>6</sup>Some recent papers that try to generate price inertia by dampening the response of marginal cost are Altig et al (2005), Neiss and Pappa (2005) and Danthine and Kurmann (2004). Huang (2006) points out that some of these mechanisms may offset each other and impede the propagation of monetary shocks.

<sup>7</sup>See Arrow (1962) and especially Rosen (1972) for early discussions of learning-by-doing as a by-product of production experience.

preferences by introducing habit formation in leisure. I discuss them in turn.

Learning-by-doing influences price inertia in two ways by providing a dynamic link between current production and future productivity. The basic mechanism is quite intuitive. As firms raise output to meet the increase in demand that follows an expansion of the money supply, they accumulate production knowledge which lowers future costs. In the periods after the shock, when firms are free to set prices, they face lower marginal costs and thus set lower prices as compared to an environment without learning-by-doing. In addition to this a more subtle mechanism may be in operation - firms may actually choose to lower their markup over marginal cost. This occurs because firms face a trade-off between increasing current revenue and reducing future marginal costs. As a result, firms use prices to control how much they learn in any given period depending on the marginal value of that learning to the firm. To see this, consider a firm that desires to reduce future costs via learning-by-doing. To do this, it must increase output. Given demand, in a monopolistically competitive environment, the firm must lower its price in order to sell this extra output.<sup>8</sup> This has two implications for the model. First, firms will set lower prices compared to standard models in which this dynamic trade-off is absent. In other words the steady state markup charged by firms is lower. Second, this markup will respond to shocks that shift the demand for the firm's product, such as a money growth shock.

Consider an increase in the growth rate of money which leads to an increase in the demand for a firm's product. This creates a favorable environment for learning because the demand curve is more responsive to a cut in price than in steady state. Essentially, a unit reduction in price yields more learning bang for the buck by generating more production and greater future cost reductions as compared to a similar price reduction in steady state. If the marginal value of learning is high for a firm, this mechanism makes it want to lower the markup it charges over marginal cost relative to steady state.<sup>9</sup> When combined with the reductions in marginal cost induced by learning-by-doing, the lower markups can be a potent mechanism for generating price inertia.

Learning-by-doing also acts as a real propagation mechanism. The short

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<sup>8</sup>I am abstracting from the possibility of inventories. Obviously a firm could, for a limited amount of time, produce output and store it rather than reduce price now. Given storage costs, this output must eventually be sold, lowering prices at that time.

<sup>9</sup>The converse is also true. If the marginal value of learning is low (perhaps because of high production in the recent past), the markup may be raised.

lived increases in output generated by a standard sticky price model in response to an unexpected increase in the growth rate of money are converted into long lived increases due to the fact that productivity is above steady state for a number of periods.

The other mechanism that contributes to the inertial response of the model is the presence of habit formation in the utility function with regards to the desire for leisure. Habits in leisure imply that consumers utility today depends not only on the current level of leisure but also past levels. Due to habit formation, high levels of leisure in the past lead to an increased desire for leisure in the present, other things being constant. To see how this might generate an inertial response in hours and output, consider the response of hours in a sticky price model in the absence of habit formation. A one time fall in the growth rate of money, leads to a reduced demand for output leading to a sharp increase in leisure as firms cut production in the impact period. The next period, prices fall and money balances are restored almost to steady state levels. As a result, output, hours and leisure revert to virtually steady state levels. When consumers form habits, the desire for leisure rises as reflected in a rise in the marginal utility of leisure in the second period. As a result, they are reluctant to return immediately to steady state levels of leisure and accordingly hours stay below steady state for a number of periods. This inertial response of hours also generates more output persistence which in turn leads to inflation persistence.

The effectiveness of these two mechanisms in generating quantitatively realistic levels of inertia in output and the aggregate price level is evaluated in the context of a dynamic general equilibrium model with real money balances in preferences. Prices, in this model, are preset for one period only. Simulations from a linearized version of the model, calibrated to the US economy, show that the model is capable of generating inertia in the aggregate price level as well as in output that is close to that observed in the aggregate US data. The first order autocorrelation coefficient of detrended output and inflation in US data is .93 and .82 respectively. The benchmark version of the model with neither habits nor learning-by-doing generates .013 and .015 respectively. The full model with both mechanisms built in delivers .71 and .81 respectively.

As far as I am aware there are no previous studies of closed economy business cycles that incorporate learning-by-doing into monetary dynamic

general equilibrium models.<sup>10</sup> However, Cooper and Johri (2002), show that learning-by-doing is extremely effective at propagating technology shocks in a real business cycle framework. Cooper and Johri use a representative agent framework and are agnostic about the issue of who actually learns from past production: workers or firms, and thus offer no account of possible decentralizations. In complementary work, Chang, Gomes and Schorfheide (2002) focus solely on learning that is embodied in workers and is fully captured in wages. They estimate the aggregate learning rate for the US using data from the PSID and incorporate this into a dynamic general equilibrium model with real shocks. They too are able to generate a persistent response of output to real shocks.

There is similarly little work on habit formation in leisure in the context of dynamic general equilibrium models with money shocks. An example is Yun (1996). Bouakez and Kano (2006) and Wen (1998) use habit formation in leisure in the context of a real business cycle model. Unlike Yun, in both papers, the stock of habit is formed based on a long lived distributed lag over past levels of leisure (or hours).

The next section presents the model and discusses the two mechanisms in more detail. Section 3 discusses the calibration of key parameters, section 4 presents some analytical and simulation results. The final section concludes.

## 2 The Model

This section describes a monetary economy populated by many identical, infinitely lived consumers. Each period, the economy finds itself in one of finitely many states,  $s^t$ . Let  $s_t = (s^0, \dots, s^t)$  be the history of these states of the world. Along with labour and a good that is used both for consumption and investment, the commodities in the economy are money, a continuum of intermediate goods, and organizational capital.

There are a large number of final good producers who behave competitively and use the following technology for converting intermediate goods

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<sup>10</sup>The closest models, Tsuruga (2007) and Cook(1999), incorporate dynamic production externalities to propagate monetary shocks. Cooper and Johri (1997) discuss how these externalities may be interpreted as learning effects. Unlike the current model, these dynamic externalities do not generate endogenous markup movements. While potentially very important, externalities are ignored in this paper to focus on the impact of internal learning-by-doing on pricing decisions.

indexed by  $i \in [0, 1]$  into final goods.

$$Y(s^t) = [\int Y_i^\eta(s^t) di]^\frac{1}{\eta} \quad (1)$$

Each period they choose inputs  $Y_i(s^t)$  for all  $i \in [0, 1]$ , and output  $Y(s^t)$  to maximize profits given by

$$\max P(s^t)Y(s^t) - \int P_i(s^{t-1})Y_i(s^t)di \quad (2)$$

subject to (1) where  $P(s^t)$  denotes the price of the final good at history  $s^t$ , while  $P_i(s^{t-1})$  is the price paid for the  $i$ th intermediate good in period  $t$ . Note that these prices are set before the realization of the period  $t$  shock. The solution to this problem gives us the input demand functions:

$$Y_i^d(s^t) = (\frac{P(s^t)}{P_i(s^{t-1})})^\frac{1}{1-\eta} Y(s^t). \quad (3)$$

The zero profit condition can be used to infer the level of final goods prices from the intermediate good prices:

$$P(s^t) = [\int P_i^\frac{\eta}{\eta-1}(s^t) di]^\frac{\eta-1}{\eta}. \quad (4)$$

There are a large number of intermediate goods producers, indexed by the letter  $i$  who operate in a Dixit-Stiglitz style imperfectly competitive economy. Each of these produces intermediate goods with a technology given by  $F(\cdot)$  which is increasing in all inputs :

$$Y_i(s^t) = F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)). \quad (5)$$

Here  $N_i(s^t)$  is the amount of labor hired,  $K_i(s^t)$  is the amount of physical capital hired by the firm to produce output,  $Y_i(s^t)$ , and  $A(s^t)$  is a common term governing the level of total factor productivity. In addition to these conventional inputs, the firm carries a stock of organizational capital,  $H_i(s^{t-1})$ , which is an input in the production technology. Organizational capital refers to the information accumulated by the firm in the process of past production regarding how best to organize its production activities and deploy its inputs.<sup>11</sup> As a result, the higher the level of organizational capital,

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<sup>11</sup> Atkeson and Kehoe (2005) model and estimate the size of organizational capital for the US manufacturing sector and find that it has a value of roughly 66 percent of physical capital.



the more productive the firm. Learning-by-doing leads to the accumulation of organizational capital which depends on output and the current stock of organizational capital:

$$H_i(s^t) = H_i^\gamma(s^{t-1})Y_i^\phi(s^t). \quad (6)$$

All producers begin life with a positive and identical endowment of organizational capital. I assume  $0 < \gamma < 1$  and  $0 < \phi \leq 1$ .

While learning-by-doing is often associated with workers and modeled as the accumulation of human capital, a number of economists have argued that firms are also store-houses of knowledge. Atkeson & Kehoe (2005) note “At least as far back as Marshall (1930, bk.iv, chap. 13.I), economists have argued that organizations store and accumulate knowledge that affects their technology of production. This accumulated knowledge is a type of unmeasured capital distinct from the concepts of physical or human capital in the standard growth model.” Similarly Lev & Radharkrishnan (2003) write, “Organization capital is thus an agglomeration of technologies—business practices, processes and designs, including incentive and compensation systems—that enable some firms to consistently extract out of a given level of resources a higher level of product and at lower cost than other firms.” There are at least two ways to think about what constitutes organizational capital. Some, like Rosen (1972), think of it as a firm specific capital good while others focus on specific knowledge embodied in the matches between workers and tasks within the firm. While these differences are important, especially when trying to measure the payments associated with various inputs, they are not crucial to the issues at hand. As a result I do not distinguish between the two.

This specification of how learning-by-doing leads to productivity increases draws on early work by Arrow(1962) and Rosen(1972) as well as a large empirical literature dating back roughly a hundred years which documents the pervasive presence of learning effects in virtually every area of the economy. Recent studies include Bahk & Gort (1993), Irwin & Klenow (1994), Jarmin (1994), Benkard (2000), Thompson (2001), Thornton & Thompson (2001) and Cooper & Johri (2002). The current specification is taken from Cooper and Johri (2002) which not only offers a detailed justification for the modelling assumptions but also a number of estimates of the learning technology at different levels of aggregation for the US economy.

The crucial difference between the traditional specification of learning-

by-doing and this one is that it allows for curvature in the accumulation of knowledge. The traditional specification assumes that organizational capital at time  $t$  equals the cumulative sum of all output ever produced by the firm. Thus it may be written as

$$H_{it} = H_{it-1} + Y_{it-1} = \sum_{j=0}^{t-1} Y_{it-j-1}. \quad (7)$$

whereas the specification used in Cooper and Johri may be written in logs as

$$H_{it} = \gamma H_{it-1} + \phi Y_{it-1} = \phi \sum_{j=0}^{t-1} \gamma^j Y_{it-j-1}. \quad (8)$$

This modification of the traditional specification of learning has a number of advantages. First, it allows for the sensible idea that production knowledge may become less and less relevant over time as new techniques of production, new product lines and new markets emerge. Second, it allows in a general way for the idea that some match specific knowledge may be lost to the firm as workers leave or get reassigned to new tasks or teams within the firm. In addition, the knowledge accumulated through production experience will be a function of the current vintage of physical capital. The decision to replace physical capital will imply that the existing stock of organizational capital will be less relevant. Third, it allows for the existence of a steady state in which the stock of organizational capital is constant. In contrast, the traditional specification in the empirical learning-by-doing literature allows the stock of organizational capital to grow unboundedly. An alternative way to bound learning is to assume that productivity increases due to learning occur for a fixed number of periods. While this may be appropriate for any one task or worker within the firm, we think of the internal context of firms as an environment with an ever changing set of tasks, workers, teams, machines and information. In this context it may be better to model organizational capital as continually accumulating and depreciating.

The restriction  $\gamma < 1$  is consistent with the empirical evidence supporting the hypothesis of depreciation of organizational capital often referred to as organizational forgetting. Argote et al. (1990) provide empirical evidence for this hypothesis of organizational forgetting associated with the construction of Liberty Ships during World War II. Similarly, Darr et al. (1995) provide evidence for this hypothesis for pizza franchises and Benkard (2000) provides

evidence for organizational forgetting associated with the production of commercial aircraft. One difference between these studies and this paper is that the accumulation technology is log-linear rather than linear. Clarke (2006) shows that the additional curvature in this log-linear technology is unlikely to produce predictions for aggregate variables, in response to a technology shock, considerably different to those associated with a linear technology. It is the implied dynamic structure associated with the accumulation of organizational capital, rather than any functional form assumptions that drives the results in Cooper and Johri (2002). Similar results should follow in the current context.

Each intermediate goods producer faces a downward sloping demand function for his product (3) which comes from the profit maximization problem of the final goods producers discussed above. Prices are set by all producers at the beginning of each period before the realization of the event  $s^t$  and cannot be changed during the period once set. Thus there are two differences between the intermediate goods firm's problem in the typical staggered price-setting model and this paper. First, the technology has been modified to incorporate learning-by-doing. Second, firms set prices for one period in a synchronized way which is a special case of an N period overlapping contracts structure with N=1.

At the beginning period t, each producer chooses a price  $P_i(s^{t-1})$  before the shocks are realized and the level of organizational capital,  $H_i(s^t)$ , after the shock is realized to maximize discounted profits:

$$\max \sum_{t=1}^{\infty} \sum_{s^t} Q(s^t | s^{t-1}) [P_i(s^{t-1})Y_i(s^t) - P(s^t)V(s^t)]$$

subject to (3) and (6) where  $Q(s^t | s^{t-1})$  is the appropriate discount rate to use to price revenue and costs in adjoining periods which is determined in the household problem.  $V(s^t)$  denotes the real cost function which has the real wage,  $w(s^t)$ , the real rental rate on capital,  $r(s^t)$ ,  $H_i(s^{t-1})$  and  $Y_i(s^t)$  as arguments. The cost function is obtained from the cost minimization exercise:

$$V_i(s^t) = \min_{N_i, K_i} w(s^t)N_i(s^t) + r(s^t)K_i(s^t) \quad (9)$$

subject to

$$F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)) \geq \bar{Y}. \quad (10)$$

The solution to this minimization problem implies

$$\frac{w(s^t)}{r(s^t)} = \frac{F_N(s^t)}{F_K(s^t)} \quad (11)$$

and

$$F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)) = \bar{Y} \quad (12)$$

from which the input demands can be obtained. Substituting these into (9) yields the cost function. Taking  $V_i(s^t)$  as given, the solution to the maximization problem above implies two first order conditions:

$$\begin{aligned} \lambda^F(s^t) = & \quad (13) \\ \sum_{s^{t+1}} Q(s^{t+1} \mid s^t) \{ & \lambda^F(s^{t+1}) \Phi'_H(H_i(s^t), Y_i(s^{t+1})) - P(s^{t+1}) V'_{Hi}(H(s^t), s^{t+1}) \} \end{aligned}$$

and

$$\begin{aligned} \sum_{s^t} Q_t[Y_{it}(\frac{-\eta}{1-\eta}) + P_t V'_{Yit}(\frac{1}{1-\eta}) \frac{Y_{it}}{P_{it}(s^{t-1})} - \\ \lambda_t^F \Phi'_Y(H_{it}, Y_{it})(\frac{1}{1-\eta}) \frac{Y_{it}}{P_{it}(s^{t-1})}] = 0 \end{aligned} \quad (14)$$

where  $\lambda^F(s^t)$  is the Lagrange multiplier associated with the organizational capital accumulation equation once (3) has been used to substitute out for  $Y_i(s^t)$ . The latter first order condition determines the optimal level of prices to be set by the producer. Note that the state has been suppressed in this equation except where it is needed to avoid confusion. Raising prices by one unit causes output to fall since producers face downward sloping demand curves for their product. The first term captures the net impact on current revenue of the higher price but lower output, while the second term represents the current cost savings from producing less. The third term appears because the producer realizes that he faces a forward looking problem due to learning by doing. The accumulation equation for organizational capital implies that a reduction of current period output will lead to a reduction in organizational capital available tomorrow. The third term captures the value

of this organizational capital lost to the firm and is made up of three parts. The term  $(\frac{1}{1-\eta})\frac{Y_{it}}{P_{it}}$  represents the reduction in output due to the higher price, while  $\Phi'_Y(H_{it}, Y_{it})$  represents the reduction in  $H_{it+1}$  due to the reduction in output which must be evaluated at  $\lambda_t^F$ , the marginal value of organizational capital to the firm.

Equation (13) determines the value of having available an additional unit of organizational capital for use by the firm in the following period. First, the additional organizational capital improves profits by reducing costs, as captured by the second term on the right hand side (recall  $V'_{Hit+1}$  is negative). Second, it adds to the ability of the organization to learn from production thus raising future organizational capital. This additional organizational capital has a value of  $\lambda_{t+1}^F$  for the firm. All this must be discounted by the price of one dollar in period  $t+1$  in units of period  $t$  dollars. Alternatively one could say that the firm sets prices so that the value of accumulating an additional unit of organizational capital today is just equal to the discounted value of organizational capital tomorrow.

The intuition in (13) and (14) suggests that firms face a trade-off between current profits and future profits which is not present in the traditional price setting problem. Charging a higher price today lowers the amount of organizational capital available tomorrow which raises future costs and lowers future profits. As a result, firms will optimally select a lower price in the presence of learning by doing than they would otherwise set. This can be seen by re-writing (14) as

$$P_{it}(s^{t-1}) = \frac{\sum_{s^t} Q_t Y_{it} (P_t V'_{Y_{it}} - \lambda_t^F \Phi'_{Y_{it}})}{\eta \sum_{s^t} Q_t Y_{it}} \quad (15)$$

and noting that the second term appears only when the learning-by-doing mechanism is present.

Note that each intermediate good firm earns positive profits even in the presence of a constant returns to scale technology due to the accumulation of organizational capital. However there is no entry or exit in this industry by assumption.

## 2.1 Consumers

The economy is populated by a large number of identical consumers whose preferences are defined over consumption of final goods ( $C(s^t)$ ), leisure ( $L(s^t)$ )

and real money balances ( $M(s^t)/P(s^t)$ ). The preference specification below allows for endogenous habit formation with  $b \geq 0$  being the parameter which determines the degree of habit persistence. These preferences reduce to the standard specification with no habits for  $b = 0$ . Each consumer maximizes the sum of discounted expected utility subject to a sequence of budget constraints (given below) by choosing the optimal quantity of these goods to consume, the amount of hours to work and how much to invest  $x(s^t)$  in physical capital ( $K(s^t)$ ) and one-period nominal bonds  $B(s^{t+1})$  each period. They take as given prices ( $P(s^t)$ ), wages ( $w(s^t)$ ) and interest rates ( $r(s^t)$ ). If  $0 < \beta < 1$  is the discount factor, then the household's problem is to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U \left( C(s^t), L(s^t) - bL(s^{t-1}), \frac{M(s^t)}{P(s^t)} \right), \quad (16)$$

subject to the sequence of budget constraints:

$$\begin{aligned} & P(s^t)C(s^t) + M(s^t) - M(s^{t-1}) + \sum_{s^{t+1}} Q(s^{t+1} | s^t) B(s^{t+1}) + P(s^t)x(s^t) \\ & \leq P(s^t) [w(s^t)N(s^t) + r(s^t)K(s^{t-1}) + T(s^t)] + B(s^t) + \Pi(s^t), \end{aligned} \quad (17)$$

and

$$N(s^t) + L(s^t) \leq 1. \quad (18)$$

Consumers lend out their stock of physical capital and labor services to intermediate goods producers and receive wages and interest income. Each of the nominal bonds,  $B(s^{t+1})$ , provides one dollar in state  $s^{t+1}$  at the expense of  $Q(s^{t+1} | s^t)$  dollars in state  $s^t$ . In addition they receive  $\Pi(s^t)$ , the current profits of intermediate goods producers as owners of all firms and  $T(s^t)$  the current real net transfers from the monetary authority. The initial conditions  $K(s^{-1})$ ,  $M(s^{-1})$ ,  $B(s^0)$  are also given. Consumers face quadratic costs of adjusting the capital stock. In particular, the capital stock evolves according to

$$K(s^t) - (1 - \delta) K(s^{t-1}) = x(s^t) - \frac{v}{2} \left( \frac{x(s^t)}{K(s^{t-1})} - \delta \right)^2 K(s^{t-1}) \quad (19)$$

While these adjustment costs are a real resource cost for the economy, since investment equals depreciation in steady state, no adjustment costs are incurred in steady state.

In addition to the above constraints, we need the sequence of borrowing constraints  $B(s^{t+1}) \geq B^u$  for some large negative value of  $B^u$ .

The first-order conditions for optimality are given by

$$U_L(s^t) = U_C(s^t)w(s^t) + b \sum_{s^{t+1}} \pi(s^{t+1} | s^t) U_L(s^{t+1}) \quad (20)$$

$$\frac{U_m(s^t)}{P(s^t)} - \frac{U_C(s^t)}{P(s^t)} = -\beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) \frac{U_C(s^{t+1})}{P(s^{t+1})} \quad (21)$$

$$Q(s^{t+1} | s^t) = \beta \pi(s^{t+1} | s^t) \frac{U_C(s^{t+1})}{U_C(s^t)} \frac{P(s^t)}{P(s^{t+1})} \quad (22)$$

$$\frac{U_C(s^t)}{1 - v \left( \frac{x(s^t)}{K(s^{t-1})} - \delta \right)} = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) \frac{U_C(s^{t+1})}{1 - v \left( \frac{x(s^{t+1})}{K(s^t)} - \delta \right)} D(s^{t+1}) \quad (23)$$

where  $U_j(s^i)$  denotes the derivative of  $U$  with respect to variable  $j$  evaluated in state  $s^i$ . Note that

$$D(s^{t+1}) = r(s^{t+1}) \left[ 1 - v \left( \frac{x(s^{t+1})}{K(s^t)} - \delta \right) \right] + 1 - \delta + \frac{v}{2} \left[ \left( \frac{x(s^{t+1})}{K(s^t)} \right)^2 - \delta^2 \right]$$

The interpretation of these first order conditions is quite standard. Equation (20) gives the optimal labor-leisure choice. The presence of habits adds a second term on the right hand side and introduces dynamics into the decision. An extra unit of leisure today generates not only some positive current marginal utility but it also raises the desire for leisure tomorrow by increasing the marginal utility of leisure tomorrow. (21) is the optimality condition determining money demand. It states that the consumer should choose to save nominal balances to the point that the current net benefit of saving an additional dollar (which is made up of the marginal utility lost due to lower current consumption minus the marginal utility gained due to higher money balances) is just equal to the discounted expected benefit next period (composed of the marginal utility of the extra consumption that can be bought next period which in turn depends on the expected value of inflation over this interval). Equation (22) is the equation which determines optimal bond holdings while equation (23) is the optimality condition for capital accumulation. This condition looks slightly different from the standard intertemporal euler equation due to the presence of adjustment costs. Since  $v \geq 0$ , at the

optimum, the household endogenizes the fact that one unit of foregone consumption today produces only  $1 - v \left( \frac{x(s^t)}{K(s^{t-1})} - \delta \right)$  units of capital tomorrow. Also, note that  $\pi(s^{t+1}|s^t) = \pi(s^{t+1})/\pi(s^t)$  is the probability of state  $s^{t+1}$  conditional on state  $s^t$  having been realized.

The nominal money supply process is

$$M(s^t) = \mu(s^t)M(s^{t-1}) \quad (24)$$

where  $\mu(s^t)$  is a stochastic process. Consumers receive lump sum transfers of new money balances which satisfy:

$$P(s^t)T(s^t) = M(s^t) - M(s^{t-1}). \quad (25)$$

In addition to these first order conditions from the consumer and firm problem we have market clearing conditions which require that the total stock of capital supplied by consumers is equal to the sum of capital rented by all intermediate goods firms. Similarly the total hours of labor supplied by consumers should equal the sum of labor hours demanded by all intermediate goods firms. Recall that while prices are chosen by firms before uncertainty about shocks is resolved, factor demands are chosen afterwards. Bond market clearing requires that  $B(s^{t+1}) = 0$ . The resource constraint for the economy is

$$C(s^t) + x(s^t) = Y(s^t). \quad (26)$$

An equilibrium is a collection of allocations for consumers,  $C(s^t), N(s^t), x(s^t), B(s^{t+1})$  and  $M(s^t)$ ; allocations for intermediate goods firms:  $N_i(s^t), K_i(s^t), H_i(s^{t-1})$  for all  $i \in [0, 1]$ ; allocations for final goods firms:  $Y(s^t), Y_i^d(s^t)$  for all  $i \in [0, 1]$ ; together with prices  $w(s^t), r(s^t), Q(s^{t+1} | s^t), P(s^t), P_i(s^{t-1})$ , for all  $i \in [0, 1]$  that satisfy the following conditions: i) taking prices as given the consumer allocations solve the consumer's problem; ii) taking all prices but its own as given, each intermediate goods producer's price and stock of organizational capital satisfies (12) and (13); iii) taking prices as given the final goods producers allocations solve the final goods producer problem; iv) the factor market conditions and resource constraint hold and the bond market clears. Only symmetric equilibria in which all consumers and producers behave identically are studied.



### 3 Computation method and calibration

The model is solved using the method outlined in King and Watson (2002) using a linear approximation to the system of equations including the first order conditions of the intermediate goods producers problem, the first order conditions from the consumers problem, the production function, the resource constraint for the economy and the accumulation equation for physical and organizational capital. Some variables are growing in steady state - they are rendered stationary by dividing by the stock of money in the economy.

In order to simulate the economy, functional forms have to be specified. With the exception of the presence of habit formation in leisure, the specification for preferences is similar to Chari, Kehoe and McGrattan (2000):

$$U(C, L, M/P) = \frac{1}{1-\sigma} [(\omega C^{(\theta-1)/\theta} + (1-\omega)(\frac{M}{P})^{(\theta-1)/\theta})^{\frac{\theta}{\theta-1}} L^{*\psi}]^{1-\sigma}. \quad (27)$$

Here  $L_t^* = L_t - bL_{t-1}$  and  $b$  governs the extent to which habits are formed. A number of estimates of  $b$  are available in the literature ranging from a high of roughly 0.8 in Eichenbaum, Hansen and Singleton (1988) to a low around 0.5 in Braun and Evans (1998). For the baseline calibration, I picked the midpoint value of  $b=.65$ . Sensitivity analysis with  $b=.5$  is also provided. Other parameters related to preferences are taken from Chari, Kehoe and McGrattan (2000). I set  $\omega = .94$ , and  $\theta = .39$  and set  $\psi$  so that the fraction of the time endowment spent on working in steady state is .3. The typical value for  $\sigma$ , in the literature is unity. However, non-separability in leisure contributes to generating inflation inertia. In order to stay close to unity and yet allow for non-separability I set  $\sigma = 1.1$ . This value of sigma was used for all models with habits in leisure including the specification referred to in the tables and figures as the full model. The autocorrelation coefficient of inflation,  $\rho_\pi$ , in the full model (but not output) is sensitive to small changes in  $\sigma$ . As  $\sigma$  goes from 1 to 1.1,  $\rho_\pi$  goes from about .6 to .8. This sensitivity to  $\sigma$  is much less acute in the absence of habits. Nonetheless higher values of  $\sigma$  deliver bigger  $\rho_\pi$  as well as less investment volatility. If  $\sigma$  is too high it is impossible to generate enough volatility in investment even without adjustment costs. Therefore for the benchmark model without learning or habits,  $\sigma$  is chosen so that the model is able to deliver the correct relative volatility of investment. This value ( $\sigma = 4$ ) is kept constant in all other cases *without habits*. In all cases, the investment adjustment cost parameter,  $v$ , is set to keep the ratio of the standard deviation of investment and output

equal to their value in the US data.

Following Cooper and Johri (2002),  $\beta$ , the discount factor is set to .984 while  $\delta$ , the depreciation rate is set to .02, the value estimated in Johri and Letendre (2007). The parameter  $\eta$  is chosen in all models to maintain a steady state markup of 5%.

Turning to the specification of technology, intermediate goods producers are assumed to use a Cobb Douglas production function to produce output, given by

$$Y_i(s^t) = N_i^\alpha(s^t)K_i^{1-\alpha}(s^t)H_i^\varepsilon(s^{t-1}). \quad (28)$$

while the accumulation equation for organizational capital is

$$H_i(s^t) = H_i^\gamma(s^{t-1})Y_i^\phi(s^t). \quad (29)$$

The elasticity of output w.r.t. physical capital is set to .39 to deliver a capital output ratio of 10.24 and  $\alpha$  is chosen to maintain constant returns in the benchmark model without learning-by-doing.

Turning to the parameters associated with learning-by-doing, I set the elasticity of output with respect to organizational capital,  $\varepsilon = 0.16$  in the specification referred to as full model. This value corresponds to a "learning rate" of just under twelve percent and is taken from 4-digit level production function estimates for US manufacturing industries provided in Cooper and Johri.<sup>12</sup> This is approximately the same value used by Atkeson and Kehoe (2005) in their study on organizational capital. Sensitivity analysis is conducted with a fifteen percent learning rule. Note that these learning rates are much lower than commonly estimated in microeconomic studies in the traditional learning-by-doing literature. The "consensus" estimate based on an extensive list of industries over the past hundred or so years appears to be twenty percent learning. See Irwin and Klenow (1994) for estimates in the semi-conductor industry as well as a discussion of past studies. Note that these studies impose quite strong restrictions on the accumulation of organizational capital which is governed by

$$H(t+1) = H(t) + Y(t).$$

In particular, note that each unit of past output contributes equally (a value of unity) to the accumulation of  $H(t+1)$  through  $H(t)$  no matter how long ago

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<sup>12</sup>See row 1 of table 2 on page 1552.

it was produced. To the extent that knowledge gleaned from past production is either lost to the firm because of labour turnover or re-organizations or becomes increasingly irrelevant over time, imposing a value of unity on the contribution of  $H(t)$  seems overly restrictive. Both Benkard (2000) and Cooper and Johri (2002) drop this restriction. Benkard shows that allowing for “organizational forgetting” leads to higher estimates of the learning rate. For example in his work on aircraft production, the estimated learning rate rises from roughly 20 percent to 39 percent once "organizational forgetting" is allowed. Keeping this in mind, and the even higher estimate of  $\varepsilon = .49$  for the aggregate economy in Cooper-Johri (2002), suggests that the amount of learning-by-doing built into the model is fairly conservative.

I set  $\gamma = .55$  and  $\phi = 1$  which are the values that correspond to  $\varepsilon = .16$ , in the estimates reported by Cooper and Johri. The persistence in the growth rate of money was set to .57, taken from Chari, Kehoe and McGrattan (2000).

## 4 Results

### 4.1 Some Analytical Results

In this section, we present some analytical results regarding the dynamics of prices implied by the learning effects introduced in this paper and compare them to some well known models in the sticky price literature. As is usual, I make some simplifications in order to solve the model analytically. I assume that there is no physical capital in the economy and impose a static money demand equation instead of (21). Habit formation in leisure is also ignored.

The key equation that governs the dynamics of prices emerges from the maximization problem of intermediate goods firms who solve the following problem.

$$\max \sum_{t=1}^{\infty} \sum_{s^{\tau}} Q(s^t | s^{t-1}) [P_i(s^{t-1})Y_i(s^t) - P(s^t)w(s^t)N_i(s^t)] \quad (30)$$

subject to (3) and (29). For our purposes, it is useful to substitute the input demand function (3) in (29) and derive an expression for  $P_i$ , the input price set by the  $i$ th firm:

$$P_i(s^{t-1}) = P(s^t)H_i^{\frac{\gamma(1-\eta)}{\phi}}(s^{t-1})Y^{1-\eta}(s^t)H_i^{\frac{-(1-\eta)}{\phi}}(s^t). \quad (31)$$

Note that the joint implication of a downward sloping demand curve and the accumulation equation for organizational capital is that prices are decreasing in  $H(s^t)$  and increasing in  $P(s^t)$ ,  $Y(s^t)$  and  $H(s^{t-1})$ . While I have discussed the reasons why prices must be reduced in order to speed up the accumulation of organizational capital, it is worth explaining why prices are increasing in  $H(s^{t-1})$ , the current level of organizational capital. The accumulation technology (29) implies that additional units of organizational capital today increase the ability of the firm to learn, leading to more organizational capital tomorrow. As a result of this improved efficiency in learning, less needs to be produced today in order to achieve any target level of organizational capital tomorrow. Given downward sloping demand curves facing firms, this implies that prices can be higher.

Assuming no capital, (28) implies

$$N_i(s^t) = Y_i^{1/\alpha}(s^t) H_i^{-\epsilon/\alpha}(s^{t-1}). \quad (32)$$

Replacing (31) and (32) in (30) and maximizing over  $H(s^t)$  yields an efficiency condition for the  $i$ th intermediate goods firm which is the equivalent of combining (13) and (14): our dynamic pricing equation. Rearranging yields the following expression:

$$\begin{aligned} \sum_{s^t} Q(s^t \mid s^{t-1}) & \left[ \frac{\eta}{\phi} P_i(s^{t-1}) \frac{Y_i(s^t)}{H_i(s^t)} - \frac{1}{\phi} P(s^t) w(s^t) H_i^{-\epsilon/\alpha}(s^{t-1}) \frac{1}{\alpha} \frac{Y_i^{1/\alpha}(s^t)}{H_i(s^t)} + \right. \\ \sum_{s^{t+1}} Q(s^{t+1} \mid s^t) & \left\{ \frac{P(s^{t+1}) w(s^{t+1}) \frac{\epsilon}{\alpha} H_i^{-(\epsilon/\alpha)-1}(s^t) Y_i^{1/\alpha}(s^{t+1}) +}{\frac{\gamma}{\phi} [-\eta P_{it}(s^t) \frac{Y_i(s^{t+1})}{H_i(s^t)} + \frac{P(s^{t+1}) w(s^{t+1}) H_i^{-\epsilon/\alpha}(s^t) Y_i^{1/\alpha}(s^{t+1})]}{\alpha H_i(s^t)}} \right\} \Big] \\ & = 0 \end{aligned} \quad (33)$$

Using symmetry in equilibrium gives us :  $Y_i(s^t) = Y(s^t)$ ,  $H_i(s^t) = H(s^t)$  and  $P_i(s^t) = P(s^t)$  for all states. Linearizing (33) around the deterministic steady state, and assuming that  $\alpha = \beta = 1$  and using (22) to substitute out  $Q$  we get:

$$\begin{aligned} \widehat{P}_t &= R_1 E(\widehat{w}_t - \epsilon \widehat{H}_t) - R_1 (\epsilon \phi + \gamma) E(\widehat{P}_t + \widehat{w}_{t+1} - \epsilon \widehat{H}_{t+1} + \widehat{Y}_t) \\ &+ (\gamma - 1) E \widehat{Y}_t + (R_1 + \gamma) E \widehat{P}_t \end{aligned} \quad (34)$$

where  $R_1 = \frac{\gamma-1}{(\gamma+\epsilon\phi)-1}$  is only a function of the three key learning-by-doing parameters. Here  $\widehat{w}_t$ ,  $\widehat{H}_t$ ,  $\widehat{P}_t$ , and  $\widehat{Y}_t$ , represent percent deviations from

steady state for  $w(s^t)$ ,  $H(s^{t-1})$ ,  $P(s^t)$  and  $Y(s^t)$  respectively and  $E$  refers to the expectation at  $t - 1$ . It is useful to compare this expression to the corresponding expression when no learning effects are present:

$$\widehat{P}_{it} = E(\widehat{P}_t + \widehat{w}_t). \quad (35)$$

which tells us that firms would change prices by the expected change in nominal wage rate.

The static money demand equation implies that  $\widehat{M}_t - \widehat{P}_t = \widehat{C}_t = \widehat{Y}_t$ . Given the preferences in (27), we can use (20) and market clearing conditions to obtain the following expression:

$$\widehat{w}_t = (1 + \chi)\widehat{Y}_t - \epsilon\chi\widehat{H}_t. \quad (36)$$

where  $\chi = (N/1 - N)\varrho$ . In this expression,  $N$  denotes steady state hours and  $\varrho$  is the elasticity of the marginal utility of leisure w.r.t. leisure. Note that in the absence of learning-by-doing, the expression would reduce to  $\widehat{w}_t = (1 + \chi)\widehat{Y}_t$  as in Chari et al (2000). Linearizing (29) gives us the following expression

$$\widehat{H}_{t+1} = \gamma\widehat{H}_t + \phi\widehat{Y}_t = \frac{\phi}{1 - \gamma L}\widehat{Y}_t. \quad (37)$$

where  $L$  is the lag operator. (37) can then be used to replace  $\widehat{H}_t$  and  $\widehat{H}_{t+1}$  while  $\widehat{w}_t$  can be replaced using (36) in (34). Now first simplify the resulting expression by noting that  $R_1(1 - (\epsilon\phi + \gamma)) = 1 - \gamma$ , and then multiplying through by  $1 - \gamma L$ , and applying the lag operator as appropriate we get our expression for output and finally replacing  $\widehat{Y}_t$  with  $\widehat{M}_t - \widehat{P}_t$  we get the expression for prices:

$$d_1 E\widehat{P}_{t+1} - d_2 E\widehat{P}_t + d_3 \widehat{P}_{t-1} = d_1 E\widehat{M}_{t+1} - d_2 E\widehat{M}_t + d_3 \widehat{M}_{t-1} \quad (38)$$

where  $d_1 = (1 + \chi)(\epsilon\phi + \gamma) + \chi$ ,  $d_2 = [(1 + \chi)(\epsilon\phi + \gamma)^2 + \chi]$  and  $d_3 = \chi\gamma + (1 + \chi)(\epsilon\phi)$ .

Comparing this expression to the equivalent expression for a two-period staggered price setting problem without learning-by-doing (taken from Chari et al in 39 below) reveals that the two equations have a similar second order difference equation structure. Apart from key differences in coefficients, which I discuss below, note also that learning-by-doing involves not only a

lead in the exogenous money process but also a lag which is missing in the absence of learning effects.

$$E\widehat{P}_{t+1} - 2\frac{2+\chi}{-\chi}\widehat{P}_t + \widehat{P}_{t-1} = -2\frac{1+\chi}{-\chi}E(\widehat{M}_{t+1} + E\widehat{M}_t) \quad (39)$$

For our preferences it is easy to check that  $\chi$  is decreasing in  $\sigma$  and in  $\psi$ . Chari et al show that the dynamics of their system is controlled by  $\chi$ .  $\chi$  also plays an important role here. While the absolute magnitude of  $d_1$ ,  $d_2$ , and  $d_3$  are all increasing in  $\chi$ , for our parameterizations these are negative numbers. As a result all three are decreasing in  $\chi$ . The dynamics of the system depends on  $\kappa_1$ , the stable root of the characteristic quadratic associated with (38) where  $\kappa_1 + \kappa_2 = \frac{d_2}{d_1}$  and  $\kappa_1 \kappa_2 = \frac{d_3}{d_1}$ . Given the values of other parameters, it is decreasing in  $\chi$  and increasing in  $\sigma$ . The stable root,  $\kappa_1$ , also depends on the learning by doing parameters. While it is increasing in all three parameters:  $\phi$ ,  $\epsilon$  and  $\gamma$ , note that it is relatively more sensitive to small changes in  $\gamma$  than the other two parameters. The degree of sensitivity is increasing in  $\sigma$ . Figure 1 plots the sensitivity of  $\kappa_1$  to these parameters.

The results of this section suggest that learning-by-doing plays an important role in increasing the inertia of the price level. Equation (38) suggests that one-period price-setting behaviour by firms mimics a two-period staggered price setting environment as in Taylor (1980) with some additional dynamics coming from the lagged money shock term. From a quantitative perspective though, the key to matching the degree of inertia seen in the data, is the value of  $\kappa_1$ . Chari et al. argue that structural restrictions placed on  $\chi$ , prevent their model from generating much inertia in either inflation or aggregate output. In Chari et al.,  $\kappa_1$  must be negative whereas, like Taylor (who treated it as a free parameter), values of  $\kappa_1$  calculated in the learning-by-doing model are positive and lie between zero and unity. From the perspective of this argument, learning-by-doing succeeds in generating realistic levels of inertia because the additional learning parameters break the tight link between  $\kappa_1$  and  $\chi$  seen in Chari et al. While, the results of this section are instructive, they are based on unrealistic restrictions which I drop in the next section.

## 4.2 Dynamics in the full model

The main question addressed in this section is how much additional inertia in the aggregate price level is generated by adding learning-by-doing and habits

in leisure to the benchmark one period sticky price model when the economy is hit by money growth shocks. I also discuss how these mechanisms influence the persistence of aggregate output movements. I begin with a discussion of the auto-correlation between inflation and its first lag ( $\rho_\pi$ ) as well as output and its first lag ( $\rho_y$ ). I then discuss the model generated impulse response of key variables to a one percent increase in the growth rate of money.<sup>13</sup>

Table 1 reports the first order auto-correlation coefficients of inflation and output for various models. Row 1 of the table corresponds to the benchmark model. In the absence of learning-by-doing and habits the one-period sticky price model delivers negligible persistence in inflation and output. The actual values are  $\rho_\pi = .015$  and  $\rho_y = .013$ . Row 2 reports corresponding moments for the full model. Both mechanisms are operational in this specification which has a learning rate of 12 percent and a moderate level of habit formation ( $b = .65$ ). There is a dramatic increase in inflation inertia and persistence in aggregate output relative to row 1. The corresponding auto-correlation coefficients rise to  $\rho_\pi = .808$  and  $\rho_y = .707$ . This compares favourably to the actual moments for US data reported in row 7 which are  $\rho_\pi = .82$  and  $\rho_y = .93$  respectively.

Rows 3-6 attempt to disentangle the contribution of the two mechanisms and provide some sensitivity analysis. Rows 3 and 4 investigate models with only learning effects but no habits while rows 5 and 6 vary the habits parameter,  $b$ , in the absence of learning-by-doing. Rows 3 and 4 show that learning-by-doing can account for a significant proportion of inflation and output inertia ranging from roughly one-third with 12 percent learning rates to roughly one-half with 15 percent learning rates. Rows 5 and 6 show that on its own, habit formation does better at accounting for inflation inertia and roughly as well at accounting for output persistence.

Figure 2 compares the impulse response of inflation between the full model and the benchmark sticky price model. Both models receive a persistent but unexpected one percent increase in the growth rate of money. Since firms cannot respond to a money shock surprise within the period, inflation is unchanged in the period of the shock. In the absence of learning or habits, firms raise prices by just over 2 percent in the period after the shock. The response of prices is much more muted in the full model, rising by roughly .3 percent in period 2. The contrast between the two models is also stark in period 3. While in the benchmark model, inflation is almost back to steady

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<sup>13</sup>The model responses to a fall in the growth rate of money are symmetric.

state levels, it rises to .65 percent in the full model and then slowly retreats back to steady state in the periods after.

The full model generates considerably more inflation inertia for two reasons. First, due to the presence of learning-by-doing and habit formation, marginal costs rise very slowly. Second, firms may wish to take advantage of the high demand for their product to learn, and therefore raise prices by less than the increase in marginal cost. In other words, firms may lower their markups. These two effects can be seen in Figures 3a and 3b. Figure 3a plots the response of nominal marginal cost in the benchmark and full model in the periods after the shock occurs. These variables are normalized by the money supply so that if costs were to rise in proportion to the money supply, the response would be zero. The behaviour of costs in the two models is quite different. In the benchmark model, nominal costs increase faster than money supply while in the full model, nominal costs increase slower than the money supply. Figure 3b plots the additional influence on price inertia exerted by time-varying markups. We see that firms lower their markup below steady state levels in the periods immediately after the shock occurs. As organizational capital is accumulated, its marginal value falls so that after three periods firms find it more profitable to raise markups slightly. A glance at Figures 3a and 3b suggest that the impact of marginal costs on price inertia is larger than that of declining markups.

Figure 4 reports the response of output, consumption, investment and hours for the benchmark and full models. In the absence of the two propagation mechanisms, all four variables jump up in the impact period and thereafter fall back to essentially steady state levels. The presence of learning-by-doing and habit formation lead to much longer drawn out responses as expected. Hours are above steady state for fourteen quarters while output is above steady state for over thirty quarters. Since the capital stock is predetermined and prices cannot be changed, firms must meet the increase in demand for their products by hiring more labour. Consumers respond to the increase in income by consuming and investing more. Since prices are fixed in the impact period, the impulse responses are similar in the two models with slightly bigger spikes in the full model. Once prices can respond to the increase in money, firms sharply increase their prices in period 2 in the benchmark model but much less so in the full model. This causes a sharp fall in real money balances in the benchmark model and therefore in the demand for intermediate and final goods. As a result, in the quarter after the shock, output, hours and investment are virtually back to their steady state levels.



Due to a slight increase in the capital stock, output is slightly above steady state levels and hours are a touch below steady state levels. Since the rise in prices is muted in the full model, the fall in real variables is also muted and elongated. The presence of learning by doing also implies that the initial spike in production leads to the accumulation of organizational capital which raises firm-level productivity in period 2. This directly leads to more production. Moreover, the higher productivity means that each firm's labour demand curve shifts outwards leading to more labour being hired in period 2 than in the benchmark model. This effect is strengthened by the habit formation process. The initial spike in hours worked leads to a fall in the marginal utility of leisure in period 2 for any level of period 2 hours. This means consumers are more willing to work at any given wage rate.

What justifies the higher production by firms in the periods subsequent to the shock in the full model? Recall that in the presence of learning effects, firms choose not to increase prices by the full increase in money. As a result, real money balances with consumers remain above steady state levels for long periods of time and consequently demand for final and intermediate goods remains high.

### 4.3 Business cycle moments

In this section I briefly ask if the improvements in the ability of the full model to generate price inertia come at the expense of performance on other fronts. Some evidence that this is not the case emerges from the impulse response analysis above. Further evidence can be gleaned by looking at a standard set of business cycle moments for the benchmark and full model relative to aggregate US data.<sup>14</sup>

Table 2 reports unfiltered theoretical second moments for the benchmark and full model in rows 1 and 2 respectively. The corresponding moments for log-linearly detrended US data (taken from Cooper and Johri (2002)) are reported in the last row. In all cases the money shock has a standard deviation of .00498 which is the value used in Nelson (1998). Looking across the rows of table 3, all the models do a good job of capturing the basic features of business cycles. Consumption, hours and investment are all procyclical and there is evidence of consumption smoothing. The two models also inherit

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<sup>14</sup>These moments emerge from an environment with only money growth shocks. Moments with both money and technology shocks are available from the author.

some common problems: in all cases consumption is more volatile than in the US data and too highly correlated with output. In fact the behaviour of consumption is virtually identical in the two models. Similarly, investment is too highly correlated with output in both models relative to the data. The first clear differences across models appear when we study the behaviour of hours. The benchmark model generates too much relative volatility compared to the data. The full model in row 2 lowers the relative volatility of hours from 1.66 to 1.34 which is still too high relative to US data. It also lowers the correlation between hours and output bringing it closer to the data.

## 5 Conclusions

Learning-by-doing and habit formation in leisure is introduced into a monetary dynamic general equilibrium model. In order to highlight the ability of the model to generate inertia in the aggregate price level, all other sources of inertia commonly used in the literature such as menu costs, staggered price or wage contracts are ignored. The model therefore relies on the minimal amount of price stickiness needed: prices are chosen before the shocks occur. A calibrated version of the model generates considerable inertia in both inflation and output dynamics in response to money growth shocks. The model also does reasonably well in matching moments that capture key features of the US business cycle.

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	$\rho_\pi$	$\rho_y$
Benchmark model: no lbd, no habits	.015	.013
Full model: 12% learning, $b = .65$	.808	.707
12 % learning, no habits	.31	.335
15 % learning, no habits	.459	.466
no lbd, moderate habits $b = .65$	.714	.511
no lbd, low habits, $b = .5$	.455	.342
US data	.8207	.9343

Table 1: Price-level inertia

	$\sigma_{C/Y}$	$\sigma_{I/Y}$	$\sigma_{H/Y}$	$\rho_{C/Y}$	$\rho_{I/Y}$	$\rho_{H/Y}$	$\rho_Y$
Benchmark: no lbd, no habits	0.92	1.30	1.64	0.99	0.99	0.99	0.01
Full model	0.93	1.30	1.33	0.99	0.99	0.92	0.71
US Data	0.69	1.30	0.52	0.89	0.60	0.71	0.93

Table 2: Second Moments

Figure 1:

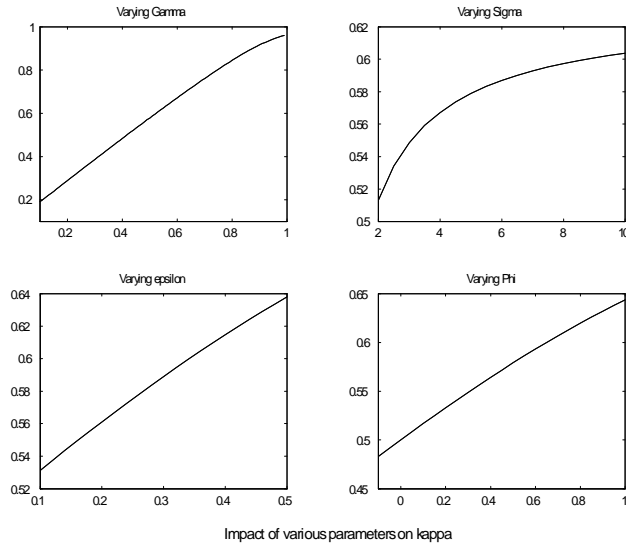


Figure2: Inflation Response

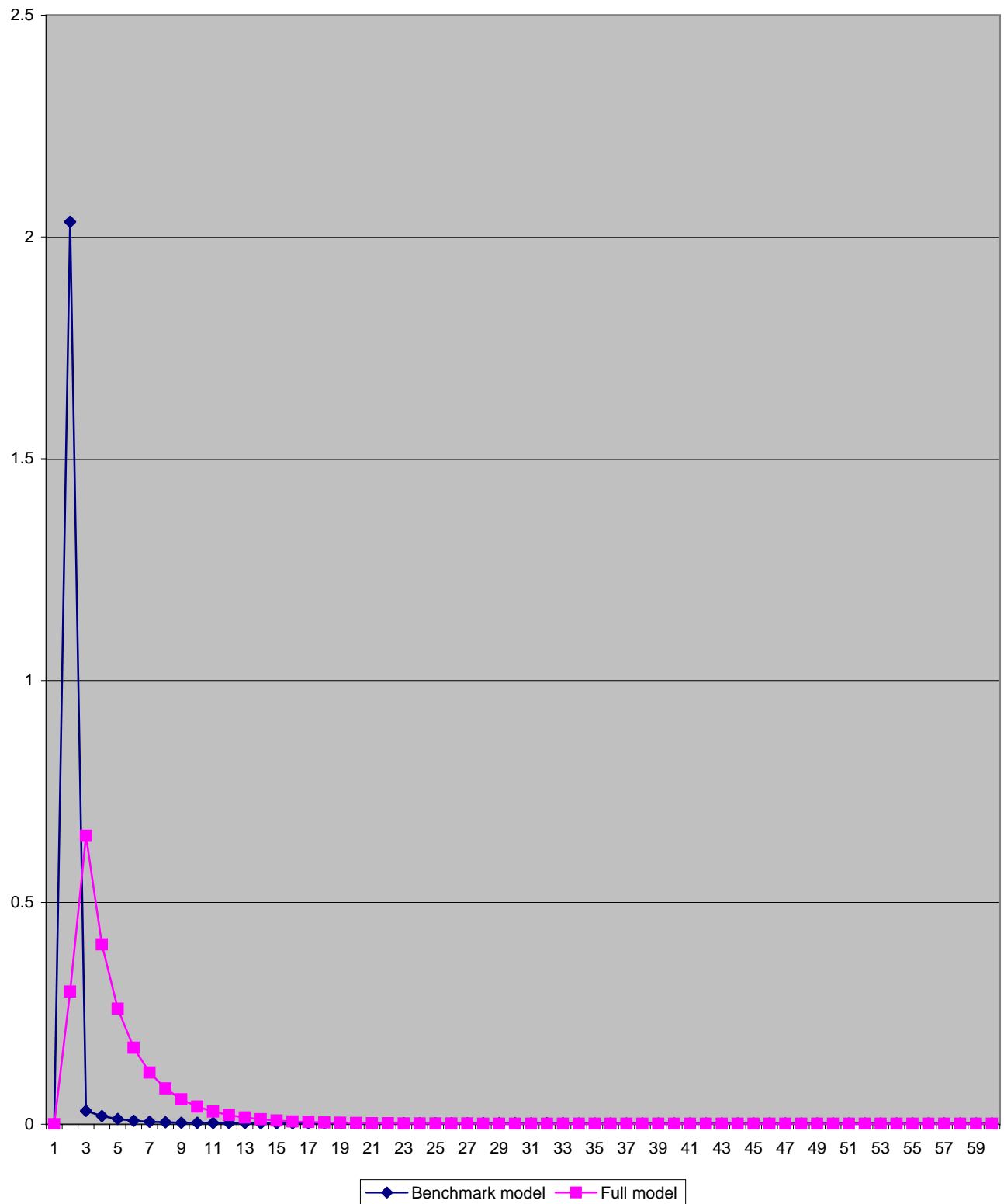




Figure 3a: Normalized marginal costs

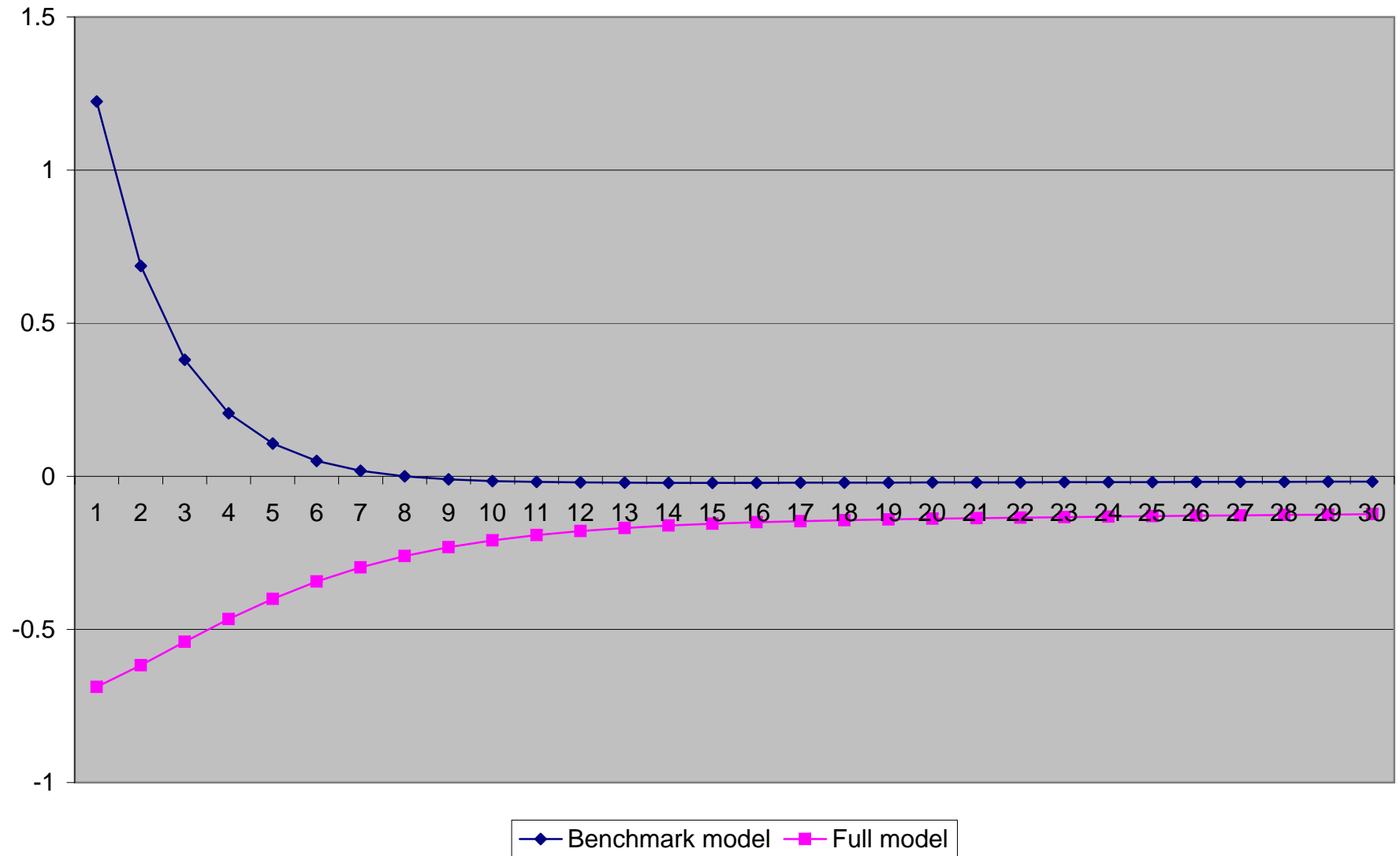


Figure 3b: Markup

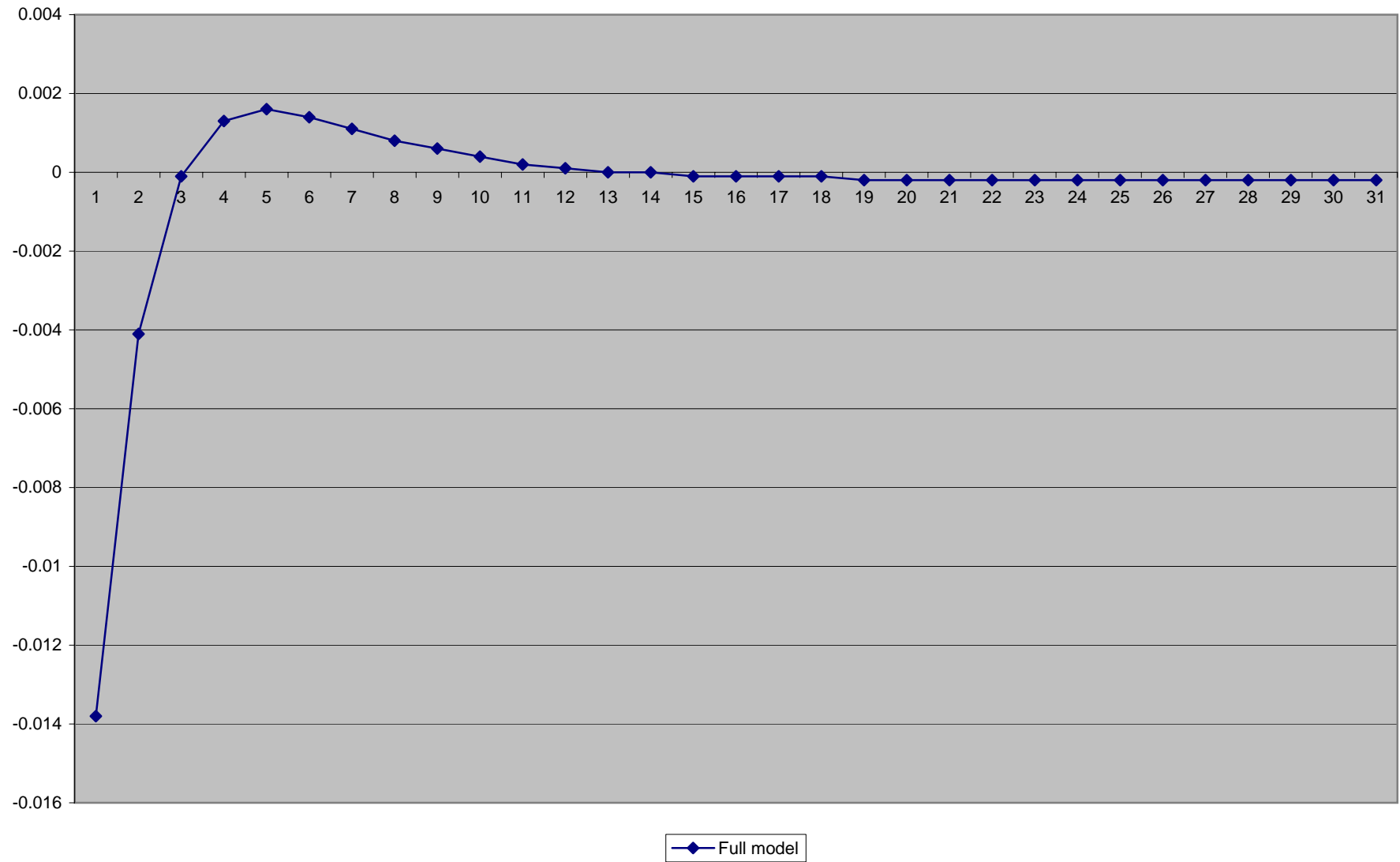


Figure 4a: Output

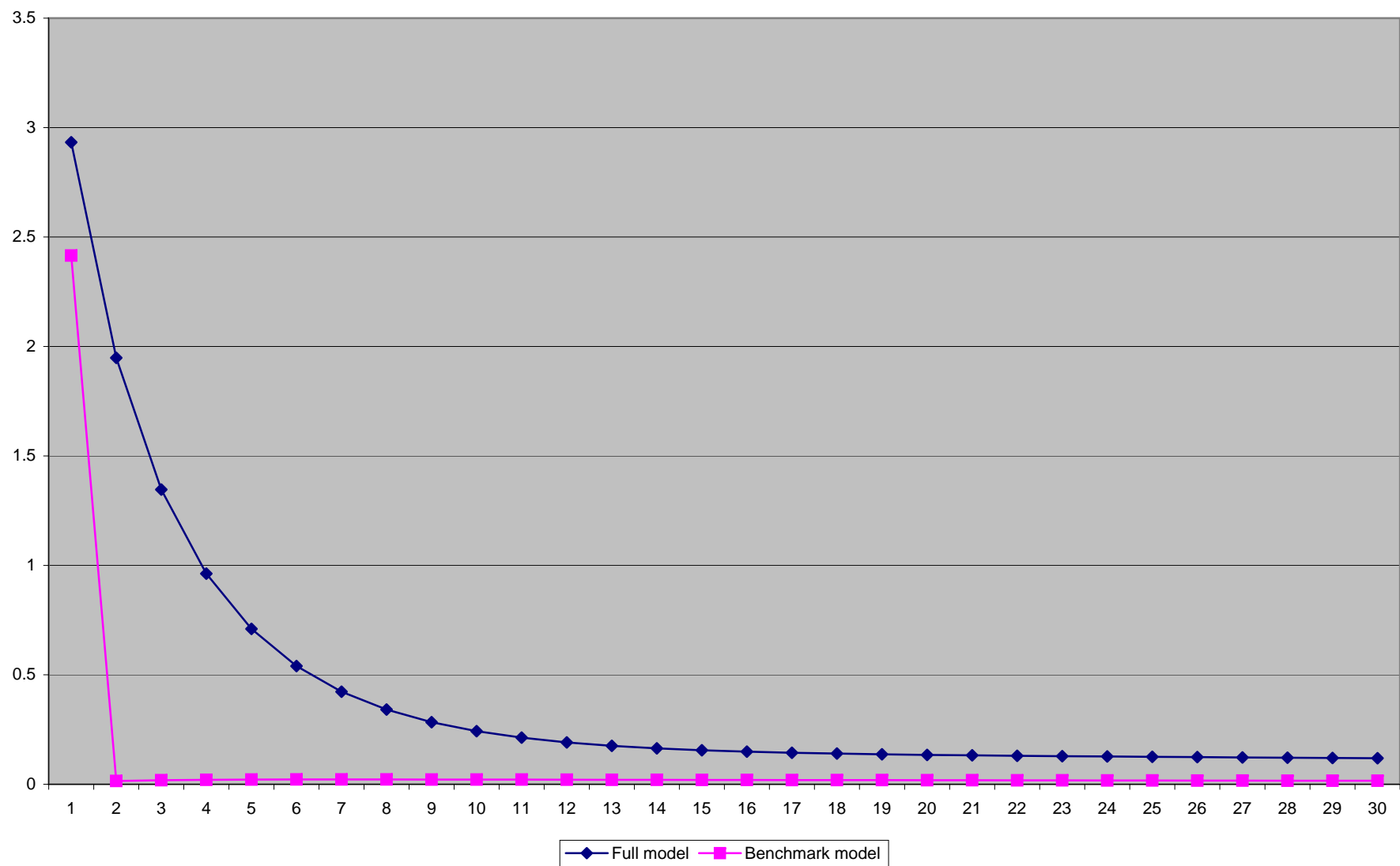


Figure 4b: Hours

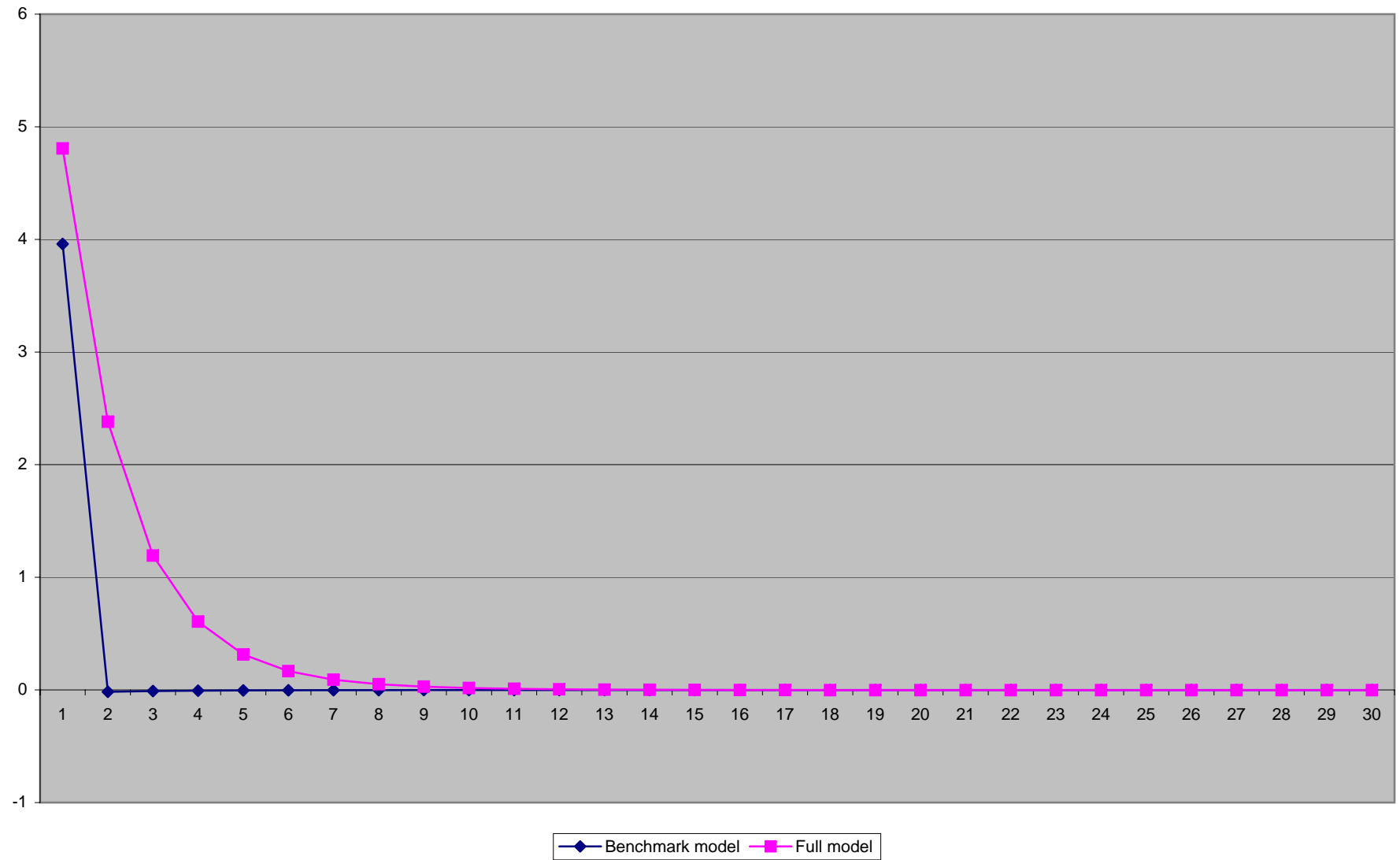


Figure 4c: Investment

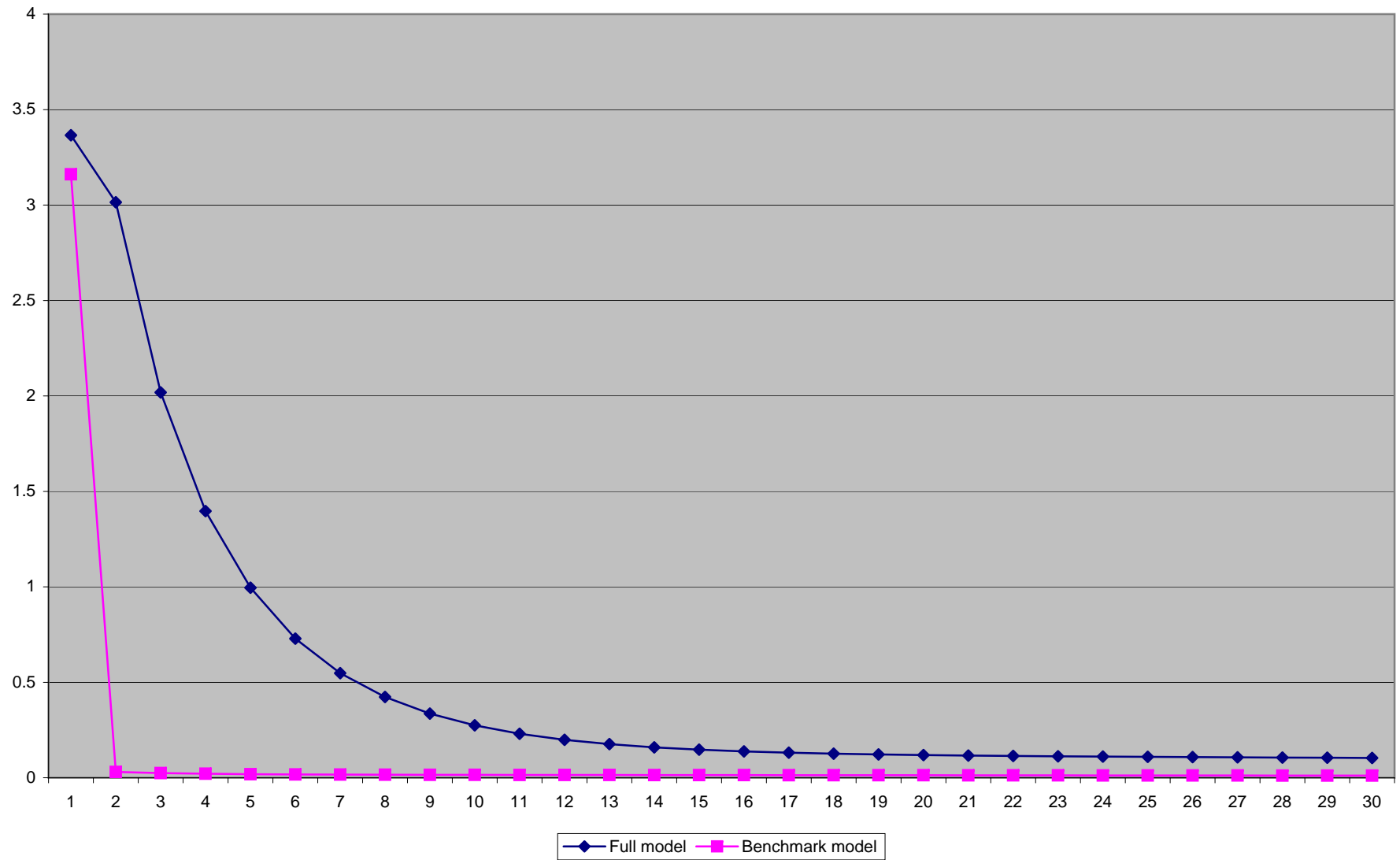


Figure 4d: Consumption

